Computing Heat Transfer Coefficients I

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Mechanical Engineering 375

Heat Transfer

March 21, 2007

Midterm Exam

- Wednesday, March 28
- Open textbook and one page equation sheet
- Will cover conduction only
  - Midterm review next Tuesday
- New material today will have self study on Monday, April 9 and quiz on Wednesday, April 11
  - Test of your memory over spring break!

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Outline

- Review last topic
- Basic pattern for computing convection coefficients
- External flows
- Classification of flows
- Flow properties
- Boundary layer
- Analytical equations

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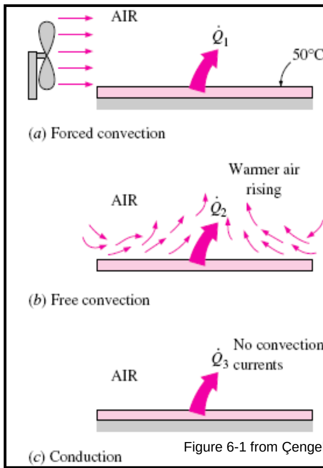
Review



- Free convection has no imposed flow field
- Forced convection does
  - may come from motion of body
- Conduction only if no fluid motion

Figure 6-1 from Çengel, *Heat and Mass Transfer*

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Review Flow Properties

- Moving fluid velocity components in x, y, and z directions are u, v, and w
- Shear stress, τ, and dynamic viscosity, μ
- For a simple flow in the x and y direction

$$\tau = \mu \frac{\partial u}{\partial y} \qquad \tau_{wall} = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

- Viscosity units kg/m·s or lb_m/ft·s
- Kinematic viscosity $\nu = \mu/\rho$ (m^2/s or ft^2/s)

Figure 6-4 from Çengel, *Heat and Mass Transfer*

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Review Flow Classifications

- Forced versus free
- Internal (as in pipes) versus external (as around aircraft)
- Unsteady (changing with time) versus unsteady (not changing with time)
- Laminar versus turbulent
- Compressible versus incompressible
- Inviscid flow region
- One-, two- or three-dimensional

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Review Dimensionless

- Nusselt number, $Nu = hL_c/k_{fluid}$
  - Different from $Bi = hL_c/k_{solid}$
- Reynolds number, $Re = \rho VL_c/\mu = VL_c/\nu$
- Prandtl number $Pr = \mu c_p/k$ (in tables)
- Grashof number, $Gr = \beta g\Delta T/\nu^2$
  - g = gravity, β = expansion coefficient = $-(1/\rho)(\partial\rho/\partial T)_p$, and $\Delta T = | T_{wall} - T_\infty |$
- Peclet, Pe = RePr; Rayleigh, Ra = GrPe

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How to Compute h

- Follow this general pattern
  - Find equations for h for the description of the flow given
    - Correct flow geometry (local or average h?)
    - Free or forced convection
  - Determine if flow is laminar or turbulent
    - Different flows have different measures to determine if the flow is laminar or turbulent based on the Reynolds number, Re, for forced convection and the Grashof number, Gr, for free convection

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How to Compute h

- Continue to follow this general pattern
  - Select correct equation for Nu (laminar or turbulent; range of Re, Pr, Gr, etc.)
  - Compute the film temperature $(T_{wall} + T_{fluid})/2$
  - Evaluate fluid properties (μ, k, ρ, Pr) at the film temperature
  - Compute Nusselt number from equation of the form $Nu = C\ Re^a\ Pr^b$
  - Compute $h = k\ Nu / L_C$

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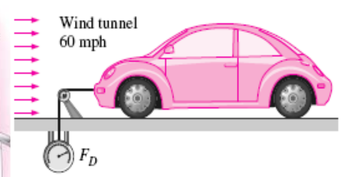
External Flow Examples



Want to determine pressure drop and heat transfer in such flows

California State University Northridge Figures 7-1 and 7-2 from Çengel, *Heat and Mass Transfer* 10

Key Ideas of External Flows

- The flow is unconfined
- Moving objects into still air are modeled as still objects with air flowing over them
- There is an approach condition of velocity, U_∞, and temperature, T_∞
- Far from the body the velocity and temperature remain at U_∞ and T_∞
- T_∞ is the (constant) fluid temperature used to compute heat transfer

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Flow Over a Flat Plate

- Idealized situation for surfaces such as an airplane wing
- Can be analyzed exactly for laminar flows
- Transition to turbulent flow exists at downstream point, x, where $Re = U_\infty x/\nu = 500{,}000$
  - $x_{transition} = x_{cr} = 500{,}000\ \nu/U_\infty$

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Flat Plate Flow Equations

- Laminar flow (Re_x, Re_L < 500,000, Pr > .6)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.664\,Re_x^{-1/2} \qquad Nu_x = \frac{h_x x}{k} = 0.332\,Re_x^{1/2}\,Pr^{1/3}$$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = 1.33\,Re_L^{-1/2} \qquad Nu_L = \frac{\bar{h}L}{k} = 0.664\,Re_L^{1/2}\,Pr^{1/3}$$

- Turbulent flow (5×10^5 < Re_x, Re_L < 10^7)

$$C_{f_x} = \frac{\tau_{wall}}{\rho U_\infty^2/2} = 0.059\,Re_x^{-1/5} \qquad Nu_x = \frac{h_x x}{k} = 0.0296\,Re_x^{0.8}\,Pr^{1/3}$$

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = 0.074\,Re_L^{-1/5} \qquad Nu_L = \frac{\bar{h}L}{k} = 0.037\,Re_L^{0.8}\,Pr^{1/3}$$

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For turbulent Nu, .6 < Pr < 60

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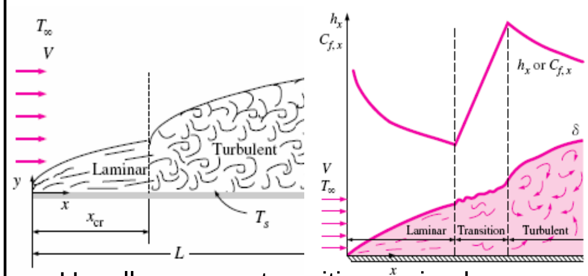
Two Views of Flat Plate Flow



- Usually assume transition region has zero thickness (as on left)

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Figures 7-6 and 7-9 from Çengel, *Heat and Mass Transfer*

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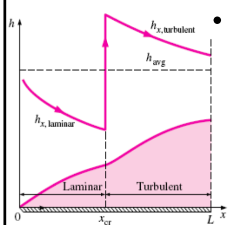
Flat Plate Flow Equations II



- Average properties for combined laminar and turbulent regions with transition at x_c = 500000 ν/U∞
  - Valid for 5×10^5 < Re_L < 10^7 and 0.6 < Pr < 60

$$C_f = \frac{\bar{\tau}_{wall}}{\rho U_\infty^2/2} = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad Nu_L = \frac{\bar{h}L}{k} = \left(0.037\,Re_L^{0.8} - 871\right)Pr^{1/3}$$

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Figure 7-10 from Çengel, *Heat and Mass Transfer*

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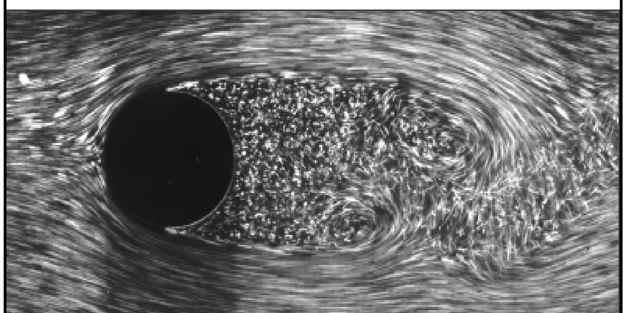
External Flow over Cylinders



Re = VD/ν = 2000

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Figure 7-16 from Çengel, *Heat and Mass Transfer*

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Drag Coefficients

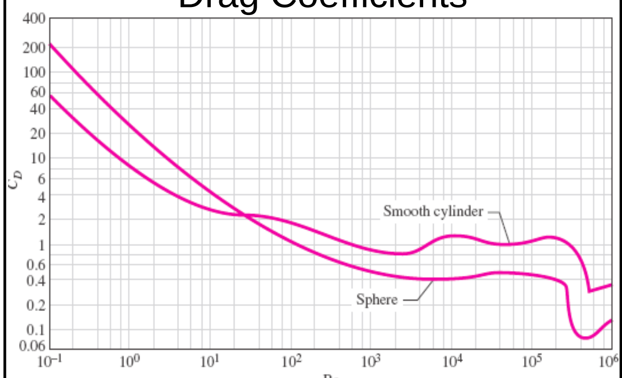

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Figure 7-17 from Çengel, *Heat and Mass Transfer*

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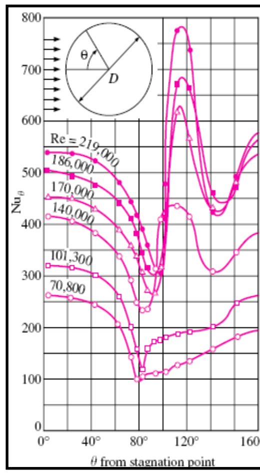

Cylinder Local h

- Stagnation point is front of cylinder (θ = 0)
- Decrease in h after θ = 0 due to thicker laminar boundary layer
- Top curves turbulent
  - Flow separation after q ≈ 140º increases h

Figure 7-22 from Çengel, *Heat and Mass Transfer*

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Heat Transfer Coefficients

- Cylinder average h (RePr > 0.2; properties at $(T_\infty + T_s)/2$

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/2}}{\left[1+\left(\frac{0.4}{\text{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

- Sphere average h ($3.5 \le \text{Re} \le 80{,}000$; $0.7 \le \text{Pr} \le 380$; μ_s at T_s; other properties at T_∞)

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

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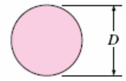
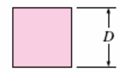
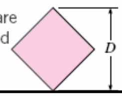
Other Shapes and Equations

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$

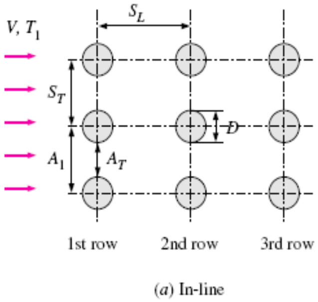
Part of Table 7-1 from Çengel, *Heat and Mass Transfer*

Flow Over Tube Banks

- Flow direction towards top
- Left: in-line
- Right: staggered

California State University Northridge Figure 7-25 from Çengel, *Heat and Mass Transfer* 21

Tube Bank Flow Notation

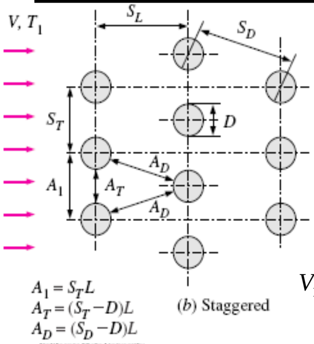


- V is inlet velocity
- S_L and S_T are lengths
- $A_1 = S_TL$, $A_T = L(S_T – D)$; L is length of tube
- $Re = V_{max}D/\nu$

$$V_{max} = \frac{VS_T}{S_T - D}$$

California State University Northridge Figure 7-26(a) from Çengel, *Heat and Mass Transfer* 22

Tube Bank Flow Notation



- Notation same as for in-line
  - New length, S_D and area, $A_D = (S_D – D)L$

$$V_{max} = \frac{VS_T}{S_T - D} \qquad 2A_D > A_T$$

$$V_{max} = \frac{VS_T}{2(S_D - D)} \qquad 2A_D < A_T$$

California State University Northridge Figure 7-26(a) from Çengel, *Heat and Mass Transfer* 23

Heat Transfer Coefficients

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	1000–2 × 10⁵	$Nu_D = 0.27\,Re_D^{0.63}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	2 × 10⁵–2 × 10⁶	$Nu_D = 0.033\,Re_D^{0.8}Pr^{0.4}(Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	1000–2 × 10⁵	$Nu_D = 0.35(S_T/S_L)^{0.2}\,Re_D^{0.6}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	2 × 10⁵–2 × 10⁶	$Nu_D = 0.031(S_T/S_L)^{0.2}\,Re_D^{0.8}Pr^{0.36}(Pr/Pr_s)^{0.25}$

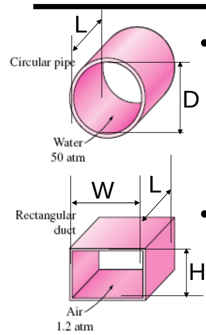
*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

California State University Northridge Table 7-2 from Çengel, *Heat and Mass Transfer* 24

Key Ideas of Internal Flows

- The flow is confined
- There is a temperature and velocity profile in the flow
  - Use average velocity and temperature
- Wall fluid heat exchange will change the average fluid temperature
  - There is no longer a constant fluid temperature like T_∞ for computing heat transfer

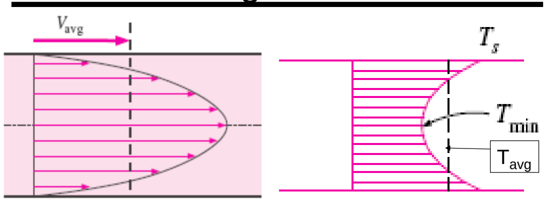
Area Terms



- A_{cs} is cross-sectional area for the flow
  - $A_{cs} = \pi D^2/4$ for circular pipe
  - $A_{cs} =$ WH for rectangular duct
- A_w is the wall area for heat transfer
  - $A_w = \pi DL$ for circular pipe
  - $A_w = 2(W + H)L$ for rectangular duct

Figure 8-1 from Çengel, *Heat and Mass Transfer*

Average Values



- Velocity and temperature profiles

$$V_{avg} = \overline{V} = \frac{\int_{A_{cs}} \rho u dA}{\int_{A_{cs}} \rho dA} \qquad T_{avg} = \overline{T} = \frac{\int_{A_{cs}} \rho u c_p T dA}{\int_{A_{cs}} \rho u c_p dA}$$

Figures 8-2 and 8-3 from Çengel, *Heat and Mass Transfer*

Average Temperature Change

- Let T represent the average fluid temperature (instead of T_{avg}, T_m or $\overline{T}$)
- T will change from inlet to outlet of confined flow
  - This gives a variable driving force ($T_{wall} - T_{fluid}$) for heat transfer
  - Can accommodate this by using the first law of thermodynamics: $\dot{Q} = \dot{m}c_p(T_{out} - T_{in})$
  - Two cases: fixed wall heat flux and fixed wall temperature

Fixed Wall Heat Flux

- Fixed wall heat flux, $\dot{q}_{wall}$, over given wall area, A_w, gives total heat input which is related to $T_{out} - T_{in}$ by thermodynamics

$$\dot{Q} = \dot{q}_{wall} A_w = \dot{m}c_p\left(T_{out} - T_{in}\right) \quad \Rightarrow \quad T_{out} = T_{in} + \frac{\dot{q}_{wall} A_w}{\dot{m}c_p}$$

- "Outlet" can be any point along flow path where area from inlet is A_w
- We can compute T_w at this point as $T_w = T_{out} + \dot{q}_{wall}/h$

Average Temperature Change

- Heat exchange with wall changes the average fluid temperature, T
- Look at small region dx with area dA_w
  - First law of thermodynamics relation for heat transfer is $d\dot{Q} = \dot{m}c_p dT$
  - Convective heat transfer: $d\dot{Q} = h dA_w(T - T_s)$
- Equating expressions for $d\dot{Q}$ gives

$$\dot{m}c_p dT = h\left(T_s - T\right)dA_w \quad \Rightarrow \quad \frac{dT}{T_s - T} = \frac{h}{\dot{m}c_p} dA_w$$

Constant Wall Temperature

- Rewrite and integrate for constant h and T_s

$$\frac{dT}{T_s - T} = \frac{h}{\dot{m}c_p} dA_w \Rightarrow \frac{d(T - T_s)}{T - T_s} = -\frac{h}{\dot{m}c_p} dA_w$$

$$\int_{T_{in}-T_s}^{T_{out}-T_s} \frac{d(T - T_s)}{T - T_s} = -\int_0^{A_w} \frac{h}{\dot{m}c_p} dA_w \Rightarrow \ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right) = -\frac{hA_w}{\dot{m}c_p}$$

- Final result for constant h and T_s

$$(T_{out} - T_s) = (T_{in} - T_s)e^{-\frac{hA_w}{\dot{m}c_p}}$$

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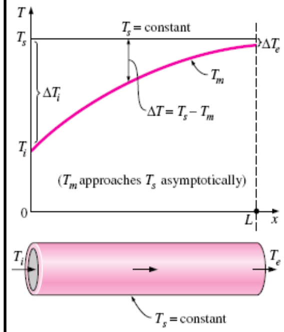
Constant Wall Temperature II



$$(T_{out} - T_s) = (T_{in} - T_s)e^{-\frac{hA_w}{\dot{m}c_p}}$$

- $hA_w/\dot{m}c_p$ = NTU, the number of transfer units
- This is general equation for computing T_{out} in internal flows

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Figure 8-14 from Çengel, *Heat and Mass Transfer*

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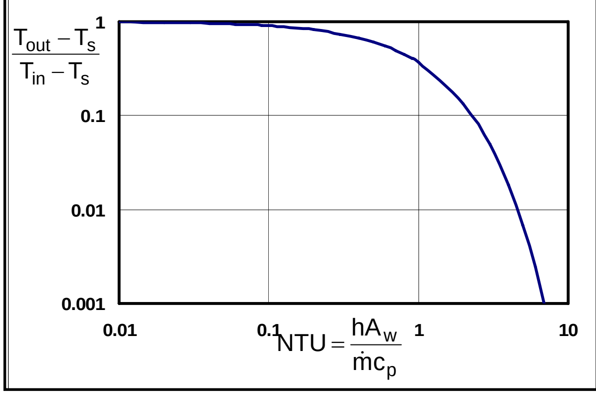
Temperature Ratio versus NTU



Constant Wall Temperature III

$$(T_{out} - T_s) = (T_{in} - T_s)e^{-\frac{hA_w}{\dot{m}c_p}} \Rightarrow \ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right) = \frac{hA_w}{\dot{m}c_p}$$

- From first law of thermodynamics $\dot{Q} = c_p(T_{out} - T_{in})\dot{m}$ so $\dot{m}c_p = \dot{Q}/(T_{out} - T_{in})$

$$\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right) = \frac{hA_w(T_{out} - T_{in})}{\dot{Q}} \Rightarrow \dot{Q} = \frac{hA_w(T_{out} - T_{in})}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)}$$

- Log mean temperature difference $LM\Delta T = \frac{(T_{out} - T_{in})}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)}$

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Figure 8-14 from Çengel, *Heat and Mass Transfer*

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Log-mean Temperature Diff

- This is usually written as a set of temperature differences

$$LM\Delta T = \frac{(T_{out} - T_{in})}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)} = \frac{(T_{out} - T_s) - (T_{in} - T_s)}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)}$$

$$\dot{Q} = \frac{hA_w(T_{out} - T_{in})}{\ln\left(\frac{T_{out} - T_s}{T_{in} - T_s}\right)} = hA_w(LM\Delta T)$$

Çengel uses ΔT_{lm} for LMΔT

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